Energy Level Statistics of the U(5) and O(6) Symmetries in the Interacting Boson Model

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Abstract

We study the energy level statistics of the states in U(5) and O(6) dynamical symmetries of the interacting boson model and the high spin states with back-bending in U(5) symmetry. In the calculations, the degeneracy resulting from the additional quantum number is eliminated manually. The calculated results indicate that the finite boson number N effect is prominent. When N has a value close to a realistic one, increasing the interaction strength of subgroup O(5) makes the statistics vary from Poisson-type to GOE-type and further recover to Poisson-type.

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However, in the case of $N \to \infty$, they all tend to be Poisson-type. The fluctuation property of the energy levels with backbending in high spin states in U(5) symmetry involves a signal of shape phase transition between spherical vibration and axial rotation.

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1 INTRODUCTION

Random-matrices theory (RMT)[1] provides a basis to study quantum chaotic systems. Particularly, the fluctuation properties of fully chaotic systems with time reversal symmetry follow the Gaussian orthogonal ensemble (GOE) whereas nonchaotic ones follow Poisson ensemble[2]. Notice that dynamical symmetry means the integrability of the system in classical limit and constants of motion associated with a symmetry govern the integrability of the system, investigating the effects of symmetry is of importance to study the dynamics of a quantum system. In recent years, many numerical studies concerning different types of symmetries and their relations to the onset of chaos have been carried out[3, 4, 5, 6, 7, 8, 9], however the case that different types of symmetries coexist and compete with each other in one quantum system has not yet been analyzed carefully. In this point of view, we study the nucleus in certain dynamical symmetries which were expected to be completely integrable in the past.

It has been known that the interacting boson model (IBM)[10] is a realistic theoretical model in describing the low-energy collective states and the electromagnetic transitions of a large number of even-even nuclei successfully. In the original version of the IBM (IBM1), nuclei are regarded as systems composed of s- and d-bosons with symmetry U(6), and it has three dynamical symmetries U(5), SU(3) and O(6), geometrically corresponding to spherical vibration, axial rotation and γ-unstable rotation[11], respectively. Because collective motion is described by a hamiltonian matrix of finite dimension, one can diagonalize the matrix easily and study the energy level statistics such as nearest-neighbor spacing distribution(NSD) numerically to check whether the motion is chaotic or regular. Then there have been many works to investigate the fluctuations of the nucleus by analyzing the energy level statistics (see, for example, Refs.[12, 13, 14]) in the framework

of the IBM. However, except for the case of SU(3) symmetry, the energy level statistics has not yet been analyzed for dynamical symmetries. Such a neglect is quite natural since, according to the symmetry paradigm, the energy level statistics should be Poisson-type. Nevertheless, the investigation on the SU(3) symmetry showed that the statistics depended strongly on the boson number of the system N and it was quite close to GOE statistics in the realistic cases where N was not very large.[6]. In this aspect, the energy level statistics of the states in a dynamical symmetry may be more complicated than the symmetry paradigm predicts. We will then analyze the energy level statistics of the U(5) and O(6) symmetries in this work. For comparison, we also involve the SU(3) limit.

More recently, a breakthrough has been carried out by Iachello in the study of critical point behavior of the nucleus undergoing a shape-phase transition. It has been shown that the critical point of the transition between vibration and γ-unstable rotation and that between vibration and axial rotation hold the symmetry E(5), X(5)[15, 16], respectively. Although fluctuation properties of these transitional regions have been studied by Alhassid and collaborates[12, 13, 14], the statistics at the critical points has not been discussed in detail. On the other hand, investigating the property of high spin nuclear states and the mechanism of backbending of high spin states has long been a significant topic in nuclear physics. It has been known that the backbending comes from the breaking of nucleon pairs and the alignment of the angular momenta. Recently, another way for the backbending, more concretely, the collective backbending to appear has been proposed to be a property of the U(5) symmetry of the IBM[17]. In such a formalism, with a special way to fix the parameters, the yrast states with the U(5) symmetry change from the vibrational ones with different d-boson numbers to the rotational ones with full d-boson configuration($n_d = N$) when the angular momentum L reaches a critical value L_c. In this sense, the energy level structure of the states in the U(5) symmetry might have a sign

of shape phase transition. We then analyze the statistics as the first step to explore the fluctuations of a shape phase transition system.

The paper is organized as follows. In Section 2, we survey the framework of the IBM and the method to analyze the energy level statistics briefly. In Section 3, we represent the numerical results and give some discussions. Finally, a summary and some remarks are given in Section 4.

2 METHOD

In the original version of the IBM (IBM1), the collective states of nuclei are described by s- and d-bosons. The corresponding dynamical group is U(6), and it has three dynamical symmetry limits U(5), O(6) and SU(3). Taking into account one- and two-body interactions among the bosons, one has the Hamiltonian of the nucleus with one of the three dynamical symmetries as[10]

$$H_{U(5)} = E_0 + \varepsilon C_{1U(5)} + \alpha C_{2U(5)} + \beta C_{2O(5)} + \gamma C_{2O(3)} \,, \tag{1}$$

$$H_{O(6)} = E_0 + \eta C_{2O(6)} + \beta C_{2O(5)} + \gamma C_{2O(3)} \,, \tag{2}$$

$$H_{SU(3)} = E_0 + \delta C_{2SU(3)} + \gamma C_{2O(3)} \,. \tag{3}$$

In case of the U(5), O(6) or SU(3) symmetry, the wave-function can be expressed as

$$|\psi_{U(5)}\rangle = |N n_d \tau K L\rangle \,, \tag{4}$$

$$|\psi_{O(6)}\rangle = |N \sigma \tau K L\rangle \,, \tag{5}$$

$$|\psi_{SU(3)}\rangle = |N(\lambda, \mu) K L\rangle \,, \tag{6}$$

where N is the total number of the bosons, $n_d, \sigma, (\lambda, \mu), \tau, L$ are the irreducible representations(IRREPs) of the group U(5), O(6), SU(3), O(5) and O(3), respectively. K is the additional quantum number to distinguish the degenerate states which have the same quantum number of the parent group.

If a nucleus is in one of the above mentioned dynamical symmetries, its energy can be given by the IRREPs as

$$E_{U(5)} = E_0 + \varepsilon n_d + \alpha n_d(n_d+4) + \beta\tau(\tau+3) + \gamma L(L+1)\,, \tag{7}$$

$$E_{O(6)} = E_0 + \eta\sigma(\sigma+4) + \beta\tau(\tau+3) + \gamma L(L+1)\,, \tag{8}$$

$$E_{SU(3)} = E_0 + \delta(\lambda^2 + \mu^2 + \lambda\mu + 3\lambda + 3\mu) + \gamma L(L+1)\,. \tag{9}$$

To analyze the energy level statistics of the states in the dynamical symmetries, we take the following process. At first, with Eqs.(7), (8) and (9), we calculate the energy levels of the nucleus in U(5), O(6) or SU(3) symmetry in IBM with different total boson number N, spin-parity J^π and several sets of parameters $\alpha, \beta, \gamma, \delta, \varepsilon, \eta$.

For a given spectrum $\{E_i\}$, it is necessary to separate it into the fluctuation part and the smoothed average part whose behavior is nonuniversal and can not be described by random-matrix theory(RMT)[1]. To do so we take the unfolding process for the energy spectrum (see for example Ref.[12]). At first we count the number of the levels below E and write it as

$$N(E) = N_{av}(E) + N_{fluct}(E)\,. \tag{10}$$

Then we fix the $N_{av}(E_i)$ semiclassically by taking a smooth polynomial function of degree 6 to fit the staircase function $N(E)$. We obtain finally the unfolded spectrum with the mapping

$$\{\widetilde{E}_i\} = N(E_i)\,. \tag{11}$$

This unfolded level sequence $\{\widetilde{E}_i\}$ is obviously dimensionless and has a constant average spacing of 1, but the actual spacings exhibit frequently strong fluctuation.

We have used two statistical measures to determine the fluctuation properties of the unfolded levels: the nearest neighbor level spacings distribution(NSD) $P(S)$ and the spectral rigidity $\Delta_3(L)$. The nearest neighbor level spacing is defined as $S_i = (\widetilde{E}_{i+1}) - (\widetilde{E}_i)$. The distribution $P(S)$ is defined as that $P(S)dS$ is the probability for the S_i to lie within the infinitesimal interval $[S, S + dS]$. It has been shown that the nearest neighbor spacing distribution $P(S)$ measures the level repulsion (the tendency of levels to avoid clustering) and short-range correlations between levels. For a regular system, it is expected to behave like the Poisson statistics

$$P(S) = e^{-S}\,, \tag{12}$$

whereas if the system is chaotic, one expects to obtain the Wigner distribution

$$P(S) = (\pi/2)S\,\exp(-\pi S^2/4)\,, \tag{13}$$

which is consistent with the GOE statistics[1, 2, 18]. With the Brody parameter ω in the Brody distribution

$$P_\omega(S) = \alpha(1+\omega)S^\omega \exp(-\alpha S^{(1+\omega)})\,, \tag{14}$$

where

$$\alpha = \Gamma[(2+\omega)/(1+\omega)]^{1/2} \tag{15}$$

and $\Gamma[x]$ is the Γ function, the transition from regularity to chaos can be measured with the Brody parameters ω quantitatively. It is evident that $\omega = 1$ corresponds to the GOE distribution, while $\omega = 0$ to the Poisson-type distribution. A value $0 < \omega < 1$ means an interplay between the regular and the chaotic.

As to the spectral rigidity $\Delta_3(L)$, it is defined as

$$\Delta_3(L) = \left\langle min_{A,B} \frac{1}{L} \int_{-L/2}^{L/2} [N(x) - Ax - B]^2 dx \right\rangle, \tag{16}$$

where $N(x)$ is the staircase function of a unfolded spectrum in the interval $[-L/2, x]$. The minimum is taken with respect to the parameters A and B. The average denoted by $\langle \cdots \rangle$ is taken over a suitable energy interval over x. Thus from this definition $\Delta_3(L)$ is the local average least square deviation of the staircase function $N(x)$ from the best fitting straight line. It has also been shown that the spectral rigidity $\Delta_3(L)$ signifies the long-range correlation of quantum spectra[2] which make it possible that for a chaotic spectrum very small fluctuation of the staircase function around its average can be found in an interval of given length (the interval may cover dozens of level spacings). For the GOE the expected value of $\Delta_3(L)$ can only be evaluated numerically, but it approaches the value

$$\Delta_3(L) \cong (lnL - 0.0687)/\pi^2 \tag{17}$$

for large L. and for Poisson statistics

$$\Delta_3(L) = L/15\,. \tag{18}$$

3 NUMERICAL RESULTS AND DISCUSSION

At first, we analyze the energy levels given in Eqs.(7) and (8) for the U(5) and O(6) dynamical symmetries, respectively. In Figs.1 and 2, we represent the NSD $P(S)$ and the Δ_3 statistics of the states with low spin-parity $J^\pi = 6^+$ in U(5) symmetry, respectively. The results at different sets of parameters (in fact, different only in parameter β) are marked with (a), (b), (c) and (d). In Figs.3(a)-(d) and 4(a)-(d), we illustrate the results

for the states $J^{\pi} = 6^{+}$ with four values of parameter β in O(6) symmetry. It has been known that the classical limit of IBM corresponds to the system with boson number $N \to \infty$. To show the finite boson number effect, we have calculated the level statistics ($P(S)$ and $\Delta_3(L)$) in each set of parameters for $N = 25$, $N = 70$ and $N = 200$. The results for these different N are displayed in the left, middle and right panel of the figures, respectively. Meanwhile the Brody parameter ω of the level spacing distribution[2] is also evaluated. The obtained results are shown in the figures. These figures show that the results in the two symmetries are quite similar to each other.

Comparing the results with the same boson number N but different parameters, one can realize that, when the boson number N has a value not very large (e.g., 25, which is close to a realistic one in nuclei), the statistics may show Poisson-type, GOE-type, intermediate between Poisson-type and GOE-type, depending on the values of the parameters. Comparing the results with different boson numbers but in one set of parameters, one can know that when $N \to \infty$, the statistics trends to be Poisson-type independently of the interaction parameters. It indicates that the finite boson number effect is prominent. Meanwhile, looking through the Figs. 1-4 more cautiously, in particular the spectral rigidity $\triangle_3(L)$ in Figs. 2(c) and 4(c), one may find that for $N = 25$ the results of the spectral rigidity lie outside the lines for Poisson and GOE since the rigidity increases very rapidly as the interval L increases. If we return to the Hamiltonian more carefully, we will find that the interaction of the parent group is much stronger than that of the subgroup, which may cause some big steps in the staircase function $N(E)$. These big steps in different quanta of the parent group can not be eliminated even after the energy levels are unfolded, and it will cause some large level spacings in the NSD $P(S)$. As these spacings are so large, they make most of the rest spacings less than 1 since the average spacing is 1. Then appears the abnormal NSD $P(S)$ statistics. For the spectral rigidity,

we extract a part of the unfolded staircase function $N(x)$ and illustrate it in Fig. 5 with a rescaling, which involves a big level spacing about 6. When L increases from 5 to 10, it is clearly that the local average least square deviation increases much more drastically than only become twice. It should also be mentioned that these abnormal statistics is in fact Poisson statistics. The looking of departure from the Poisson-type statistics is because the number of these large level spacings which affect strongly the statistics is not very large(less than N). So they distribute randomly in the NSD $P(S)$ and do not follow the statistical principles. If we do not consider these large abnormal spacings, the level spacings which are less than 1 will shift to following Poisson statistics as the unfolded constant will decrease. The spectral rigidity will not increase so fast as they are left out.

It should be mentioned that, in our calculations, all the degenerate states are taken into consideration just as one single state. That means, if the quantum numbers of some states differing from others only in an additional quantum number, we take the energy levels of these states as one single level when the energy level statistics is carried out. On the contrary, if we regard them as distinctive levels, the degeneracy causes so many zero level spacings that the distribution is over-Poisson type. Fig. 6 displays the results for the states $J^{\pi} = 6^{+}$ of the $N = 25$ system with the degenerate states being taken into account distinctively. Comparing Figs. 1-4. with Fig. 6, one can easily recognize that the difference between the results with and without the degeneracy being considered explicitly is apparent. In practical calculation, nearly 1/4 levels of all are abandoned when we have chosen just one level out of each set of the degenerate states. It is obvious that such a manual selection of the levels introduces a finite symmetry breaking to lift the degeneracy.

It has been known that the degeneracy results from the absence of sufficient quantum numbers due to symmetry. In previous numerical calculations where the transitions from one dynamical symmetry to the other were investigated, since the symmetries have been

broken, the degeneracy is then broken, such a problem seems do not exist. However, when we analyze the statistics in the dynamical symmetries, we have to handle the problem since the additional quantum number K in Eqs.(4-6) can be quite large if the boson number N is large. In the previous investigations on the statistics of the energy levels in SU(3) symmetry[5, 6], Paar and collaborators discussed the case of $J^{\pi} = 0^{+}$, where the additional quantum number K takes only one value $K = 0$, and also the case of $J^{\pi} \geq 2^{+}$, where K could have more than one values. Such an additional quantum number K may be viewed as a result of a hidden symmetry[19] since the states with the same angular momentum but different K are degenerate. The calculated results showed that, for $N = 20$ which is not very large, the statistics of the states $J^{\pi} = 0^{+}$ is close to GOE-type. As the boson number N increases, the statistics gets close to Poisson-type. For the states $J^{\pi} = 2^{+}$, if the K is fixed to a certain number, the energy level statistics is closed to GOE-type. In the present work, we also analyze the case of SU(3) symmetry with different boson numbers. In the analysis we select only one level from each set of degenerate states to establish the level set for statistics, which is just the same as that taken for the U(5) and O(6) symmetries, and is equivalent to that with $K \equiv 0$ in Paar's work. The obtained results for the states of $J^{\pi} = 6^{+}$ in the systems with boson number $N = 25$, 70 and 200 are illustrated in Fig. 7. The calculated results show that the trend of statistics from GOE-type to Poisson-type as N increases is clear, which coincides with the result of Paar and collaborators[6].

For the sake of a self consistent discussion, we also show explicitly the energy level statistics of the states $J^{\pi} = 0^{+}$ for which the degeneracy caused by hidden symmetry does not exist. Since numerical results show that the statistics of the O(6) symmetry is quite close to that of the U(5) symmetry, we represented then only the results for the $P(S)$ and $\Delta_3(L)$ statistics in U(5) dynamical symmetry in Figs. 8 and 9, respectively. It

is clear that the energy level statistics is quite similar to that in Figs. 1 and 2, as we have discussed before. It manifests that our removal by hand of all but one of the degenerate states is equivalent to that without multi-degeneracy .

As mentioned above, in order to show how the manually introduced symmetry breaking affects the statistics, we also calculate the statistics with distinctive degenerate states. The results for the case with $N = 25$ are shown in Fig. 6. Comparing Fig. 6 with Figs. 1-4 in the case of the same parameters, one can easily infer that the manually introduced symmetry breaking makes the statistics from over-Poisson type to GOE type. The results are quite consistent with Paar and collaborators' work[5], where they introduce an additional term which breaks the K quantum number but conserves SU(3) dynamical symmetry. Their results show that increasing the strength of the K breaking term makes the statistics change continuously from over-Poisson type to GOE type. One thing we need then to point out here is that in practical calculation, the hidden symmetry is completely broken not in the case that the degeneracy resulting from the existence of additional quantum number is removed(types of the interaction), but in the case that the strength breaking the symmetry reaches a certain value(strength of the interaction). This might interpret why in Alhassid and collaborates' work[12], the statistics near the dynamical symmetries is in an "overintegral" situation with negative Brody parameter ω. When the Hamiltonian they use become very close to the one in the dynamical symmetry, for instance, when $c_0 = 0$ and $\chi = -0.01$ in the self-consistent Q formalism[20] near the O(6) dynamical symmetry, the broken strength is too weak to break the hidden symmetry resulting from the missing labels though the degeneracy does not exist. Then the question comes out that which way to determine the level set can better describe the statistics of the realistic nucleus in the dynamical symmetry. In the present O(5)$\supset$ O(3) reduction, the degeneracy due to the hidden symmetry is distinguished by the manually introduced ad-

ditional quantum number but no interaction is involved to link the states with different additional quantum number K. Therefore the degenerate states with different additional quantum numbers are in fact statistically uncorrelated. When we calculate the fluctuations of the energy levels, such a large amount of the statistically uncorrelated states should be removed. Otherwise, a mixed ensemble (with different good quantum number K) is taken into consideration and the over-Poisson type distribution would be obtained (because nearly 1/4 of the spacings of all are zero in practical calculations). In this point of view, the "overintegral" situation in Ref.[12] may arise from that some statistically uncorrelated energy levels were taken into consideration.

In Fig. 7, we display the results with only one set of parameters $\gamma = 0.01$, $\delta = -0.7$ (in arbitrary unit), because from Eq.(9), one can know that different values of parameters γ and δ cause only a linear transformation of the energies. Then it does not affect the statistics. Analogously, one may get a conclusion from Eqs.(7) and (8) that changing the parameter should not affect the statistics in U(5) and O(6) symmetries since the type of the interaction and the structure of the energy levels remain the same. However, recalling Figs. 1-4, one can realize that, if the boson number is not very large(e.g. N=25), the statistics in U(5) or O(6) symmetry depends obviously on the absolute value of the parameter β. This indicates that the relative strengths of the interactions with different symmetries also affect the statistics. If we take the results more carefully, we will find that the increase of the relative value β (in fact β/α, β/η) makes the statistics in realistic case ($N = 25$) change gradually from the Poisson-type to GOE-type. In order to show the dependence of the statistics on the interaction strengths more obviously, we calculate the Brody parameter ω of the level spacing distribution in a wide range of parameter β in U(5) and O(6) symmetries, respectively. The obtained results are given in Figs. 10(a),(b), respectively. The figures show that the statistics varies from Poisson-type to GOE-type,

and further to Poisson-type again with respect to the increasing of β/α, β/η. Recalling Eqs.(7) and (8), we can realize that for a small value of β/α, β/η, the interaction with the O(5) symmetry is only a perturbation on the U(5), O(6) symmetries, the quantum system is then approximately regular. While the ratio increases, the interaction strength of the O(5) becomes comparable to the strength of the parent group U(5) or O(6), the statistics appears in GOE-type. It indicates that, when the strengths of the interactions with different symmetries are comparable and compete with each other in one quantum system, chaos may come out. This mechanism of onset of chaos can also be seen in Alhassid and collaborates' work of investigating the broken pairs in nuclei[21], where when the Coriolis interaction is comparable to the pairing interaction, the degree of chaoticity seems to be maximal. As the ratio of the interactions changes further and becomes so large that the interaction of the parent group plays only a role as a "perturbation", the quantum system recovers approximately regular.

It is worth mentioning that the above results are quite similar to the well-know case of the hydrogen atom in a uniform magnetic field[22]. The Sturm-Coulomb problem is an integrable one since it holds O(4) symmetry. When one puts the atom into a magnetic field, the O(4) symmetry is broken and reduced to the O(2) symmetry. The problem becomes then nonintegrable. The chaos arises and is being obvious when the energy or the magnetic strength (connected to O(2) strength) increases. The present U(5)$\supset$ O(5) and O(6)$\supset$ O(5) reductions are analogous to the O(4)$\supset$ O(2). The onset of chaos in the U(5) and O(6) symmetry is a direct result against the corresponding increase of the interaction strength of O(5) symmetry since the symmetry is, in fact, broken.

Aside from the above analogy, another problem might have some relation with the above results. In the geometric analysis of IBM, the subgroup O(5) in the dynamical algebra U(6) corresponds only to the kinetic part of the Hamitonian[23], which can be

written as

$$T_\gamma = p_\gamma^2 + \frac{1}{4}\sum_{m=1}^{3}\frac{L_m^2}{\sin^2(\gamma - 2\pi m/3)} \tag{19}$$

in the classical limit. As a result, the O(5) symmetric term can be viewed as the kinetic interaction that does not affect the $\beta-\gamma$ dependence of the potential surface and contains only the collective motion of the nucleus according to Leviatan and collaborates' work[24]. Indeed, the interaction strength of the subgroups in the system, such as that of the O(5) symmetric one in the IBM, can be viewed as parts of the dynamical origin of the chaotic behavior, or more concretely, the GOE fluctuations in nuclei. Just as Bohigas pointed out[25] :"In our opinion, the static nuclear mean field is too regular to be held responsible, and chaos must be caused by the residual interaction." In this point of view, the onset of chaos in nucleus results not only from the symmetry breaking of the potential(mean field), but also from the increasing of the interaction strength of the subgroup(residual interaction), which is competing with the parent group.

Furthermore, Eq.(7) shows that the energy of the states in U(5) symmetry depends not only on parameter β, but also on the other parameters, such as α and ε. Considering the geometric model correspondence of the IBM, one knows that U(5) symmetry corresponds to an inharmonic vibration with frequency $\hbar\omega = \varepsilon + (n_d+4)\alpha$. It is obvious that, with the increasing of the d-boson number, if $\alpha > 0$, the vibration frequency increases; if $\alpha < 0$, the vibration frequency decreases. It has recently been shown that the U(5) symmetry with parameter $\alpha < 0$ can describe the collective backbending of high spin states well[17]. For a given set of parameters with $\alpha < 0$, there exists a critical angular momentum

$$L_c \approx -\frac{2(\varepsilon + 4\alpha)}{\alpha} - 2N\,, \tag{20}$$

where N is the total boson number. As the angular momentum $L \geq L_c$, the yrast states are no longer the inharmonic vibrational states, but the rotational ones with $n_d = N$. For

a system with N=25 and parameters $\varepsilon = 3.53$, $\alpha = -0.101$(in arbitrary unit), in which L_c=12, we analyze the energy level statistics of the states $L = 6(< L_c)$, $L = 12(= L_c)$ and $L = 20(> L_c)$. The obtained results are illustrated in Fig. 11. It is apparent that two maxima appear in the nearest neighbor level spacing distribution $P(S)$. Such a behavior is quite different from the fluctuation properties in other cases. In theoretical point of view, the term $n_d\varepsilon$ enlarges the level spacing, whereas the term $n_d(n_d + 4)\alpha$ with $\alpha < 0$ compresses the level spacing. The simultaneous appearance of these two effects induces a competition which makes the energy of the states in the ground state band of the U(5) symmetry

$$E_{gsb}(n_d) = (\alpha + \beta + 4\gamma){n_d}^2 + (\varepsilon + 4\alpha + 3\beta + 2\gamma)n_d \tag{21}$$

not increase with respect to the increasing of the d-boson number monotonously. Then a maximal d-boson number limit $n_d^{(m)}$ for the energy to increase correspondingly exists

$$n_d^{(m)} = \frac{\varepsilon + 4\alpha + 3\beta + 2\gamma}{-2(\alpha + \beta + 4\gamma)} \approx \frac{\varepsilon + 4\alpha}{-2\alpha}\,. \tag{22}$$

Considering the property of the parabola $E_{gsb}(n_d)$, one can know that there exists also a boson number $n_d^{(u)} = 2n_d^{(m)} - N$, with which the energy of the system equates to that with $n_d = N$, and those with $n_d \in (n_d^{(u)}, n_d^{(m)})$ are larger than those of the states with the same angular momentum but $n_d = N$. It means that the states in the intrinsic ground state band with d-boson number $n_d \in (n_d^{(u)}, N)$ are no longer the yrast states. Then the structure of the yrast band changes from the U(5) states to the rotational states with d-boson number $n_d = N$ and the energy of the states in the yrast band changes in the way $L(L + 1)$. It implies that a phase transition of collective motion mode may happen as the angular momentum reaches the critical value ($L_c = 2n_d^{(u)}$) in Eq.(20).

We can also speculate it more clearly in the thermodynamical analysis. If we use the coherent state formalism[11, 26] of the IBM, the energy functional for the U(5) Hamilto-

nian is given by

$$E(N, f_1, f_2; \beta) = E_0 + f_1 N \frac{\beta^2}{1+\beta^2} + f_2 N(N-1) \frac{\beta^4}{(1+\beta^2)^2}. \tag{23}$$

The parameter of the first term f_1 depends on the parameter $\varepsilon, \beta, \gamma$ used in Eqs.(1) and (7), while the parameter f_2 of the second term depends only on the parameter α. It is definite that if the interaction shown in the second term is attractive, which corresponds to $\alpha < 0$ and $f_2 < 0$, there exist two minima for the energy functional in Eq. (23). One is E_0 arising from $\beta = 0$. Another is $E_0 + f_1 N + f_2 N(N-1)$ corresponding to $\beta \to \infty$. In the general framework of the Landau theory of phase transitions[27], it has been well known that, for a system with thermodynamic potential $\Phi(P, T; \xi)$ that depends on external parameters (pressure P and temperature T) and the order parameter ξ, the first-order transitions are characterized by a singularity in the specific heat $C_p = -T\partial^2\Phi/\partial T^2$, which gives a nonzero latent heat. Meanwhile, the optimal order parameter ξ_0 to minimize the functional jumps *discontinuously* from one value to another. It has been well known that the deformation parameter β is the quantity to characterize the shape of a nucleus, it can then be taken as the order parameter to identify the nuclear shape transition. Then, instead of the thermodynamic potential $\Phi(P, T; \xi)$, the $E(N, f_1, f_2; \beta)$ in Eq. (23) can be taken as the "thermodynamic potential"[28, 29]. Analogous to the above mentioned general case, the *discontinuous* jump of the minima of the energy functional $E(N, f_1, f_2, \beta)$ against the order parameter β (from 0 to ∞) indicates that the shape phase transition discussed here is in first-order. This is in agreement with the results found for the vibron model[30, 31] with random interactions, which exhibits a first order phase transition between a ground state with $n_p = 0$ and $n_p = N$[32]. It is also remarkable that the rotation after the shape phase transition is the one with variable-momentum since the stable state corresponds to the asymptotic process $\beta \to \infty$.

It has been known that, as a general rule, phase coexistence can be found in first-order phase transitions[27]. In our present energy level statistics analysis, the appearance of different shapes makes the ensemble (with the same angular momentum L) in the U(5) symmetry with $\alpha < 0$ involves in fact two sequences, one of which is in vibration, another one is in rotation. When we unfold the above spectrum $\{E_i(L)\}$, the two sequences are normalized with an unique total average spacing, and their maxima of the spacing distributions do not appear at the same value of S, As a consequence, two maxima emerge. It should be noted that there are only few overlaps between the two different sequences, otherwise the overlap of the two sequences will change the nearest neighbor distribution. On the other hand, recently two new symmetries denoted by E(5)[15] and X(5)[16, 33] have been proposed to describe the spectra of nuclei at or around the critical point of shape transitions from vibration to γ-unstable rotation (second-order) and those from vibration to axial rotation (first-order), respectively. And the X(5) symmetry describes the nuclei at or around the critical point of the first-order shape phase transition which involves shape coexistence[34, 35]. For comparison, We also analyze the energy level statistics of the states with E(5) and X(5) symmetries. Our results show that the NSD $P(S)$ of X(5) symmetry exhibits also two maxima(the full result will be published elsewhere[36]). Such a similarity in the NSD might provides further evidence that the collective backbending is a manifestation of shape phase transition in first order and involving shape coexistence.

To manifest the constituent of the level ensemble, we evaluate the rigidity $\Delta_3(L)$ more meticulously, too. The result is illustrated in Fig. 12. Fig. 12 shows that the $\Delta_3(L)$ of the spectrum with collective backbending exhibits a drastic fluctuation. It has been known that the variance of the Δ_3 statistics ($\langle\Delta_3^2\rangle - \langle\Delta_3\rangle^2$) is connected with the 3- and 4-level correlations, which is expected to be very small in the past[1]. The conflicts between the obtained results and general behavior indicts that the system undergoing a shape phase

transition might exhibit a much strong fluctuations than the usual systems discussed in the past.

For comparison, we evaluate the statistics of the system with $N = 70$, 200 for $\alpha < 0$, too. The calculated results for $L = 6$ states are represented in Fig. 13. One can know from Eq.(20) that, for N=70 or 200, $L_c < 0$ if ε and α maintain their values as the same as those for N=25, the competition mentioned above does not play any role for all the states. Then the two maxima in the NSD statistics and the drastic fluctuation in the Δ_3 statistics no longer exist. The appearance of the Poisson-type distribution indicates that only the rotational mode plays important role in the system. Comparing the result with N=25 and those with N=70, 200, one can reach a conclusion that the appearance of the collective backbending is a signal of phase transition from a vibration to a rotation. Meanwhile the emergence of two maxima in the $P(S)$ distribution may be a characteristic of the shape phase transition.

4 SUMMARY AND REMARKS

In summary, we have analyzed the energy level statistics of the U(5), O(6) and SU(3) dynamical symmetries in the interacting boson model(IBM) in this paper. In the analysis, the degeneracy resulting from the additional quantum number was eliminated manually, i.e. we took only one level from each set of degenerate states. The calculated results indicate that the finite boson number N effect is prominent. If N takes a value not very large(e.g., 25, which is close to a realistic one), the statistics depends strongly on the interaction strength of the subgroup O(5). While $N \to \infty$, they all trend to be Poisson-type. We would like then to mention that the interaction of the subgroup O(5) which only possess the collective motions of the nucleus can be viewed as the dynamical origin of

chaos in nucleus and the symmetry paradigm deserves more careful consideration. In fact, exceptions to the symmetry paradigm have been found for many years (see, for example, Ref.[37]).

In this paper, we also analyzed the level statistics of the states holding the collective backbending in high spin states. We found that the nearest neighbor level spacing distribution $P(S)$ of the states with the collective backbending involved two maxima and the Δ_3 statistics exhibited a fierce fluctuation, which were drastically different from the general properties in each symmetry of the IBM at usual situation. It indicates that the spectrum involves a shift between two modes of collective motions. It provides then a clue that the collective backbending is a characteristic of shape phase transition. Furthermore, looking through all the process, we can suggest that the statistics of the system can result from not only the form of interaction (the Hamiltonian or perturbation) but also the interaction strength.

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Figures and Their Captions:

Fig. 1. Comparison of nearest-neighbor spacing distribution $P(S)$ of the states $J^\pi = 6^+$ in U(5) symmetry with four sets of parameters: (a) for $\varepsilon = 1.76$, $\alpha = 0.1$, $\beta = 0.02$, $\gamma = 0.001$, (b) for $\varepsilon = 1.76$, $\alpha = 0.1$, $\beta = 0.01$, $\gamma = 0.001$, (c) for $\varepsilon = 1.76$, $\alpha = 0.1$, $\beta = 0.006$, $\gamma = 0.001$, (d) for $\varepsilon = 1.76$, $\alpha = 0.1$, $\beta = -0.01$, $\gamma = 0.001$. In all figures, the solid lines and dashed lines describe the GOE and Poisson statistics, respectively.

Fig. 2. The same as Fig. 1 but for the spectral rigidity $\Delta_3(L)$.

Fig. 3. Comparison of nearest-neighbor spacing distribution $P(S)$ of the states $J^\pi = 6^+$ in O(6) symmetry with four sets of parameters: (a) for $\eta = -0.5$, $\beta = 0.15$, $\gamma = 0.001$, (b) for $\eta = -0.5$, $\beta = 0.10$, $\gamma = 0.001$, (c) for $\eta = -0.5$, $\beta = 0.05$, $\gamma = 0.001$, (d) for $\eta = -0.5$, $\beta = -0.10$, $\gamma = 0.001$.

Fig. 4. The same as Fig. 3 but for the spectral rigidity $\Delta_3(L)$.

Fig. 5. Comparison of the local average least square deviation to a part of staircase function $N(E)$ (solid steps). The levels analyzed are the $J^\pi = 6^+$ levels of the same U(5) Hamiltonian as Fig. 1(c) with boson number $N = 25$. The dash lines and dashed-dotted lines describe the local average least square lines as the interval L equates 5 and 10, respectively.

Fig. 6. Energy level statistics for U(5) and O(6) symmetries with distinctive degenerate states caused by additional quantum number in the states $J^\pi = 6^+$ when boson number $N = 25$. (a) For U(5) symmetry with parameters $\varepsilon = 1.76$, $\alpha = 0.1$, $\beta = 0.01$, $\gamma = 0.001$. (b) For O(6) symmetry with parameters $\eta = -0.5$, $\beta = 0.15$, $\gamma = 0.001$.

Fig. 7. Energy level statistics for the $J^\pi = 6^+$ states in SU(3) symmetry with different number of bosons.

Fig. 8. The same as Fig. 1 but for angular momentum and parity $J^\pi = 0^+$.

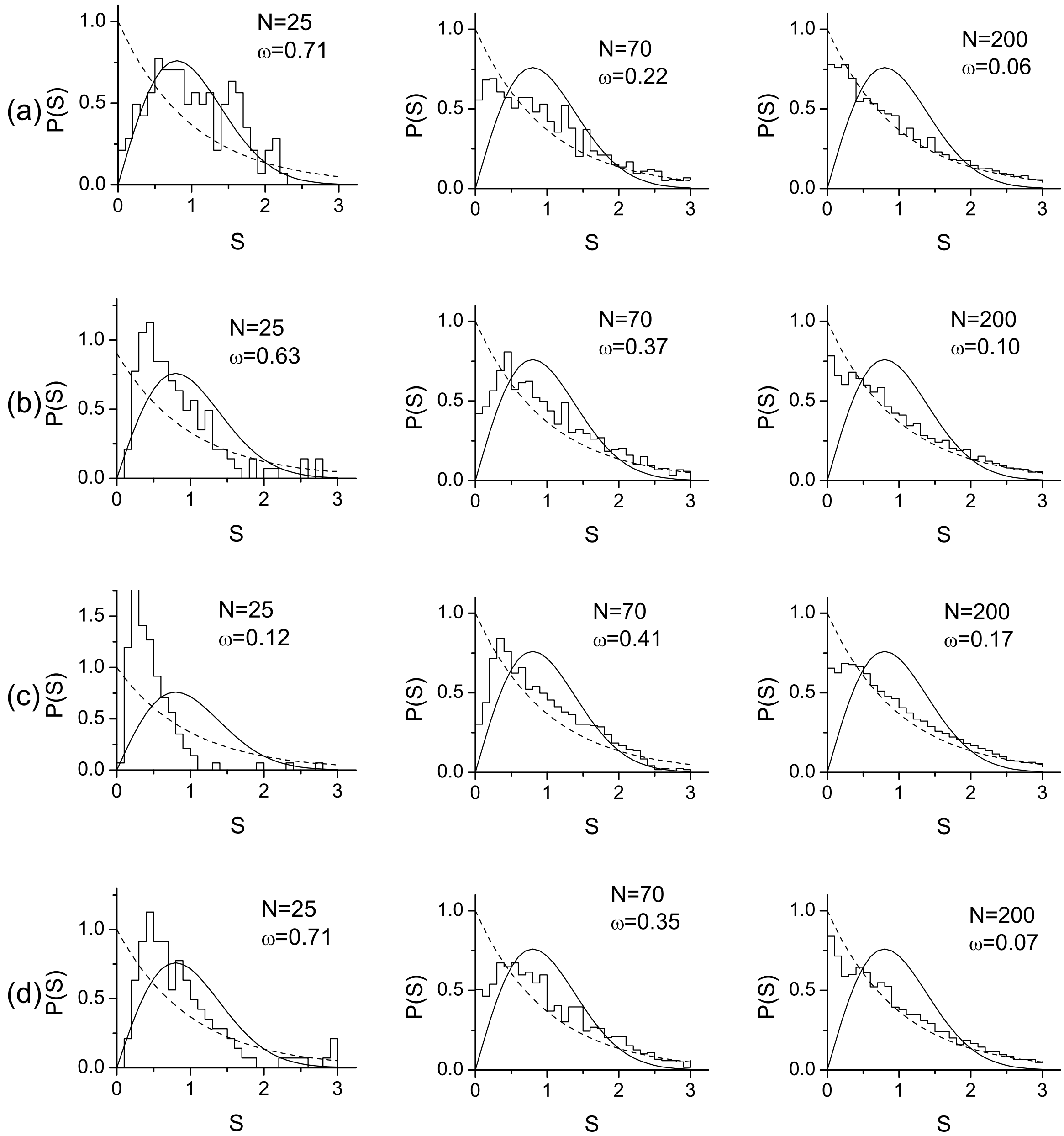
Fig. 9. The same as Fig. 2 but for angular momentum and parity $J^\pi = 0^+$.

Fig. 10. Quantum measures of chaos in different interaction strength of O(5) symmetry: (a) Brody parameter ω versus β/α for $\varepsilon = 1.76$, $\alpha = 0.1$, $\gamma = 0.001$ in U(5) symmetry, (b) Brody parameter ω versus β/η for $\eta = -0.5$, $\gamma = 0.001$ in O(6) symmetry.

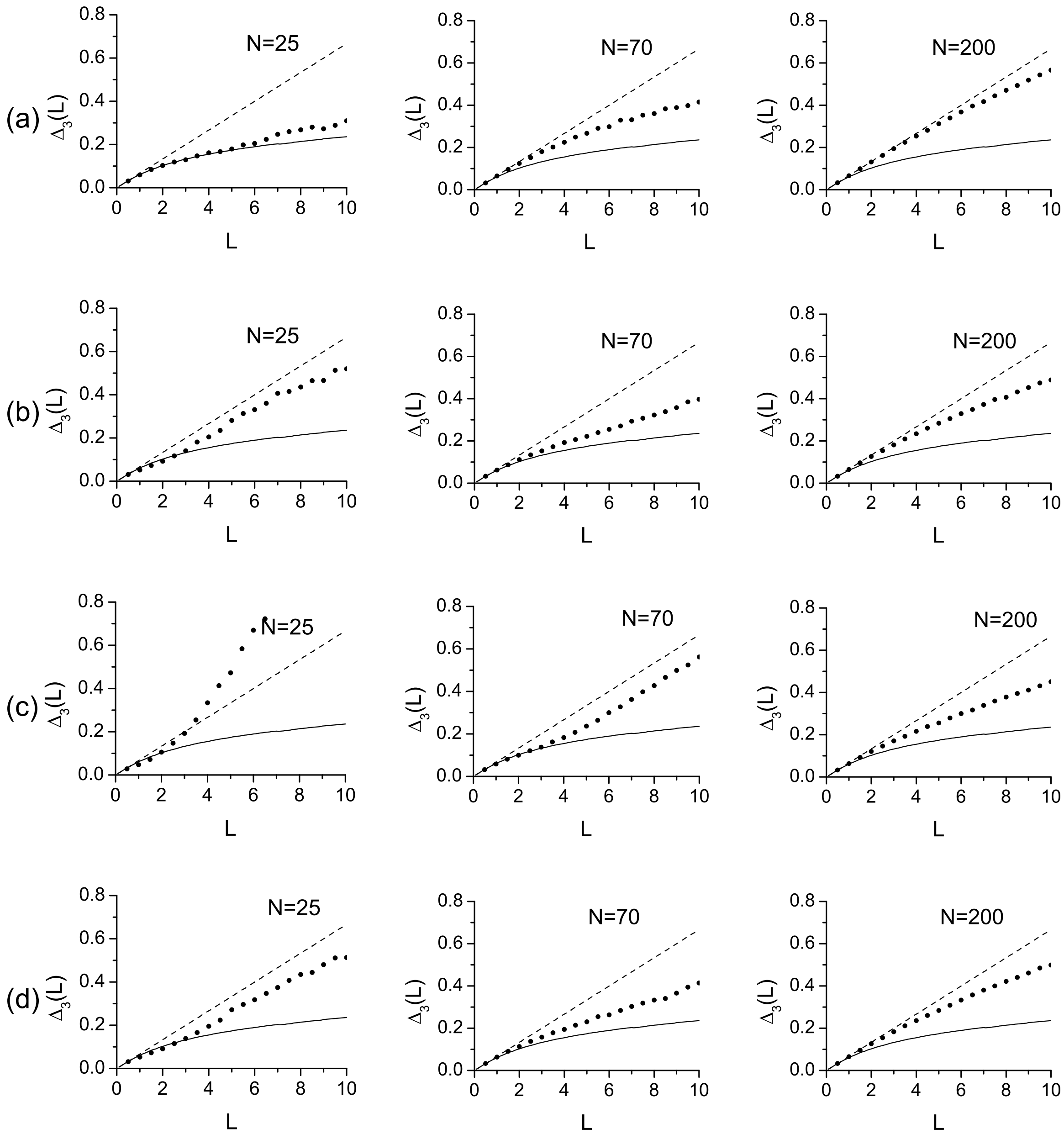
Fig. 11. Energy level statistics of the states with boson number $N = 25$, angular momentum and parity $J^\pi = 6^+(L < L_c)$, $12^+(L = L_c)$, $20^+(L = L_c)$ in U(5) symmetry when it has collective backbending at high spin states (The parameters are $\varepsilon = 3.51$, $\alpha = -0.101$, $\beta = 0.01$, $\gamma = 0.001$).

Fig. 12. Comparison of Δ_3 statistics of the states in U(5) symmetry with boson number $N = 25$ with collective backbending and without collective backbending: (a) for $\varepsilon = 3.51$, $\alpha = -0.101$, $\beta = 0.01$, $\gamma = 0.001$, $J^\pi = 6^+(L < L_c)$, (b) the same parameters with (a) but for $J^\pi = 12^+(L = L_c)$, (c) the same parameters with (a) but for $J^\pi = 20^+(L = L_c)$, (d) for $\varepsilon = 1.76$, $\alpha = 0.1$, $\beta = 0.02$, $\gamma = 0.001$, $J^\pi = 6^+$.

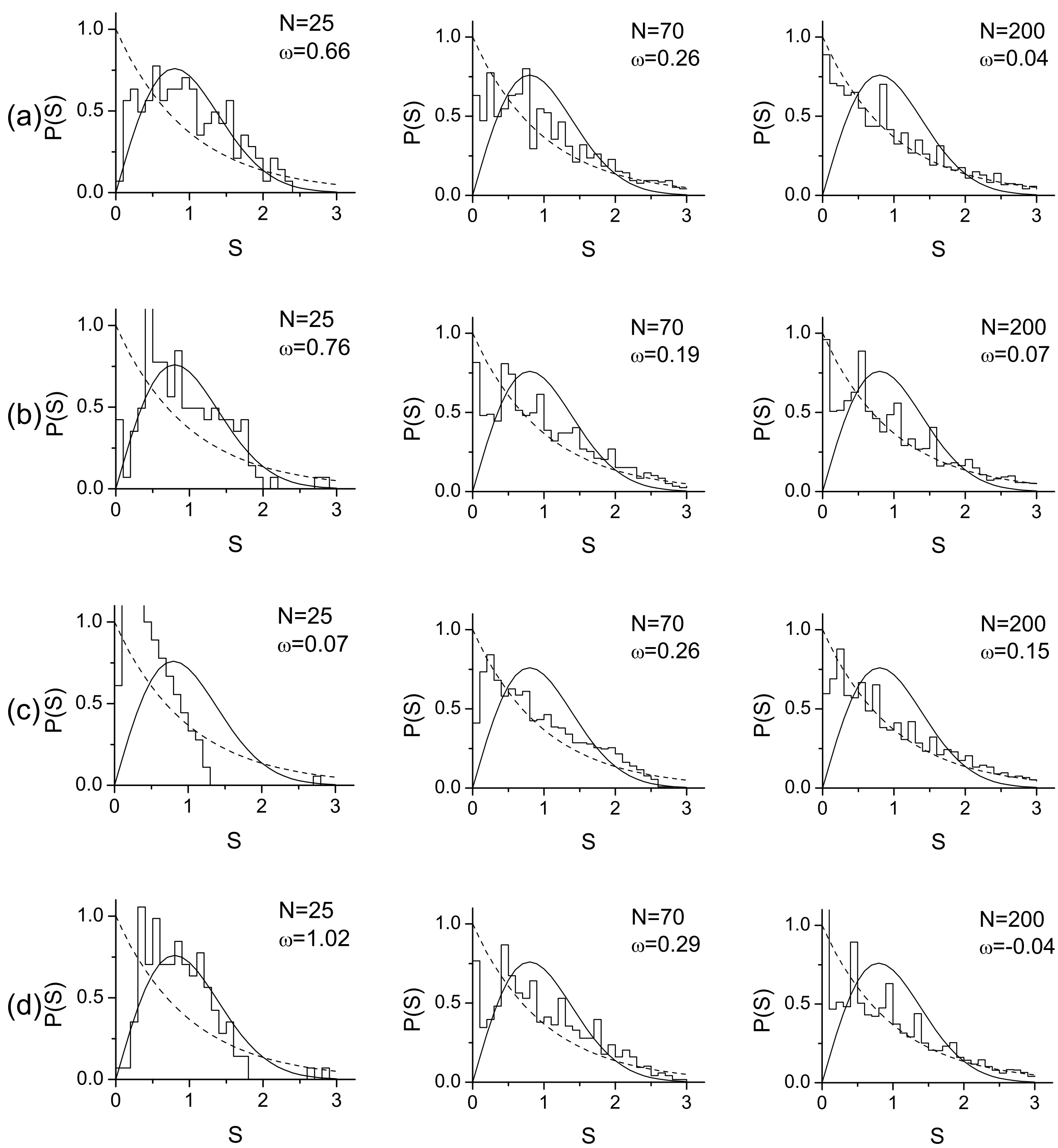
Fig. 13. The same as Fig. 11 but with boson number $N = 70$ and 200 for the $J^\pi = 6^+$ states.

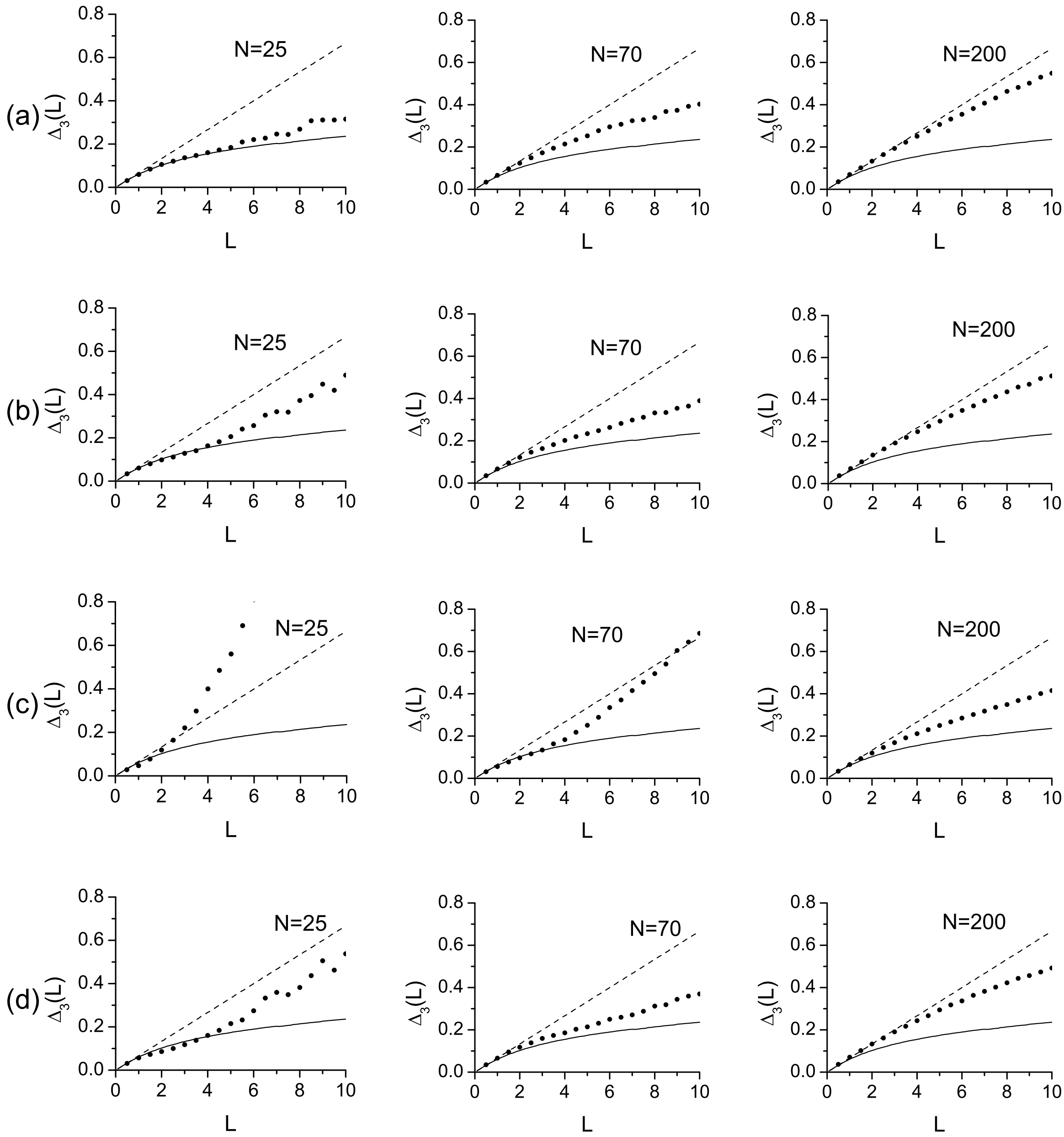


(a)
(b)
(c)
(d)
N=25
ω=0.71
N=70
ω=0.22
N=200
ω=0.06
N=25
ω=0.63
N=70
ω=0.37
N=200
ω=0.10
N=25
ω=0.12
N=70
ω=0.41
N=200
ω=0.17
N=25
ω=0.71
N=70
ω=0.35
N=200
ω=0.07
P(S)
S

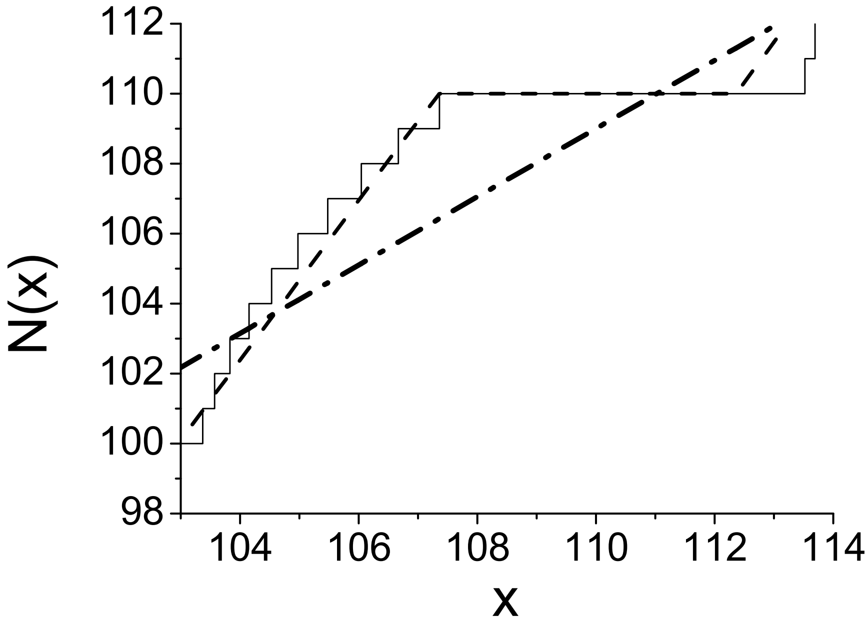


(a)
(b)
(c)
(d)
N=25
N=70
N=200
Δ3(L)
L
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
10

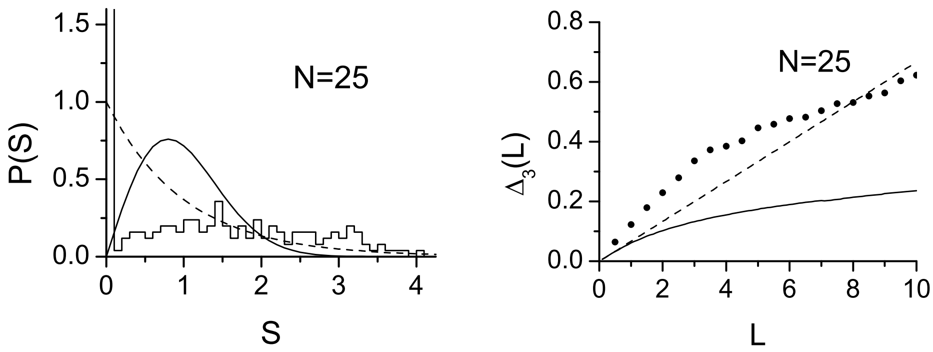

(a)
N=25
ω=0.66
N=70
ω=0.26
N=200
ω=0.04
(b)
N=25
ω=0.76
N=70
ω=0.19
N=200
ω=0.07
(c)
N=25
ω=0.07
N=70
ω=0.26
N=200
ω=0.15
(d)
N=25
ω=1.02
N=70
ω=0.29
N=200
ω=-0.04
P(S)
S
0.0
0.5
1.0
0
1
2
3



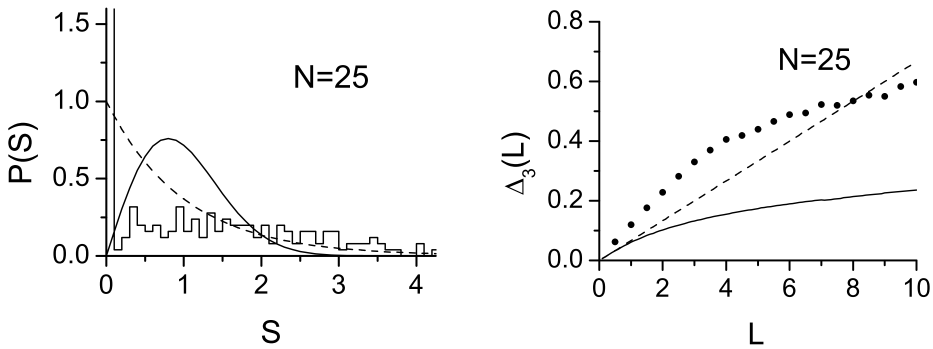
(a)
(b)
(c)
(d)
N=25
N=70
N=200
$\Delta_3(L)$
L



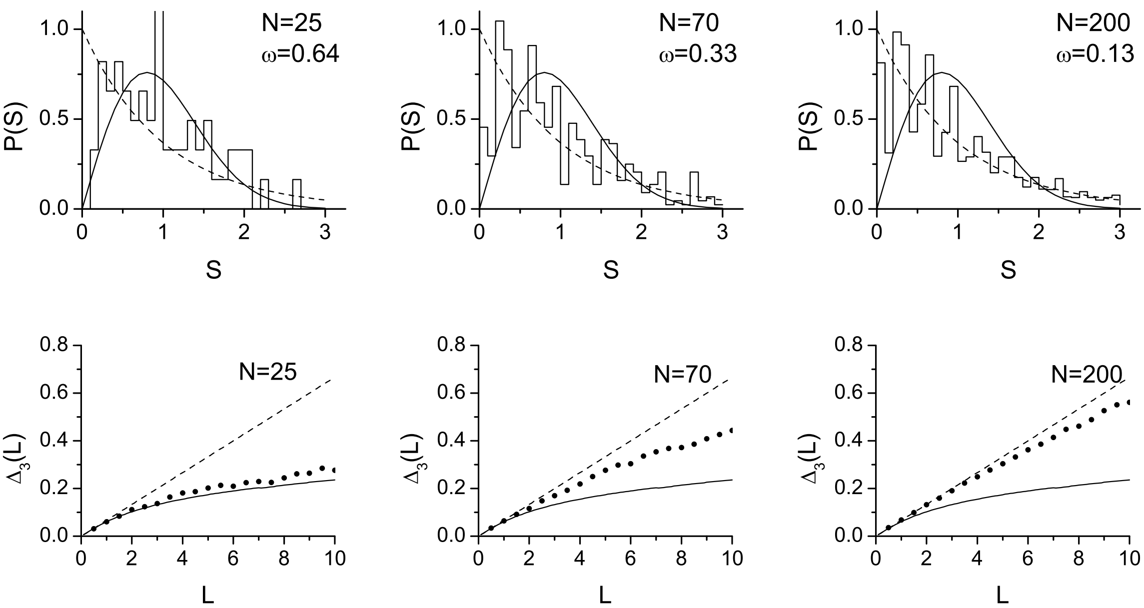
112
110
108
106
104
102
100
98
N(x)
104
106
108
110
112
114
x

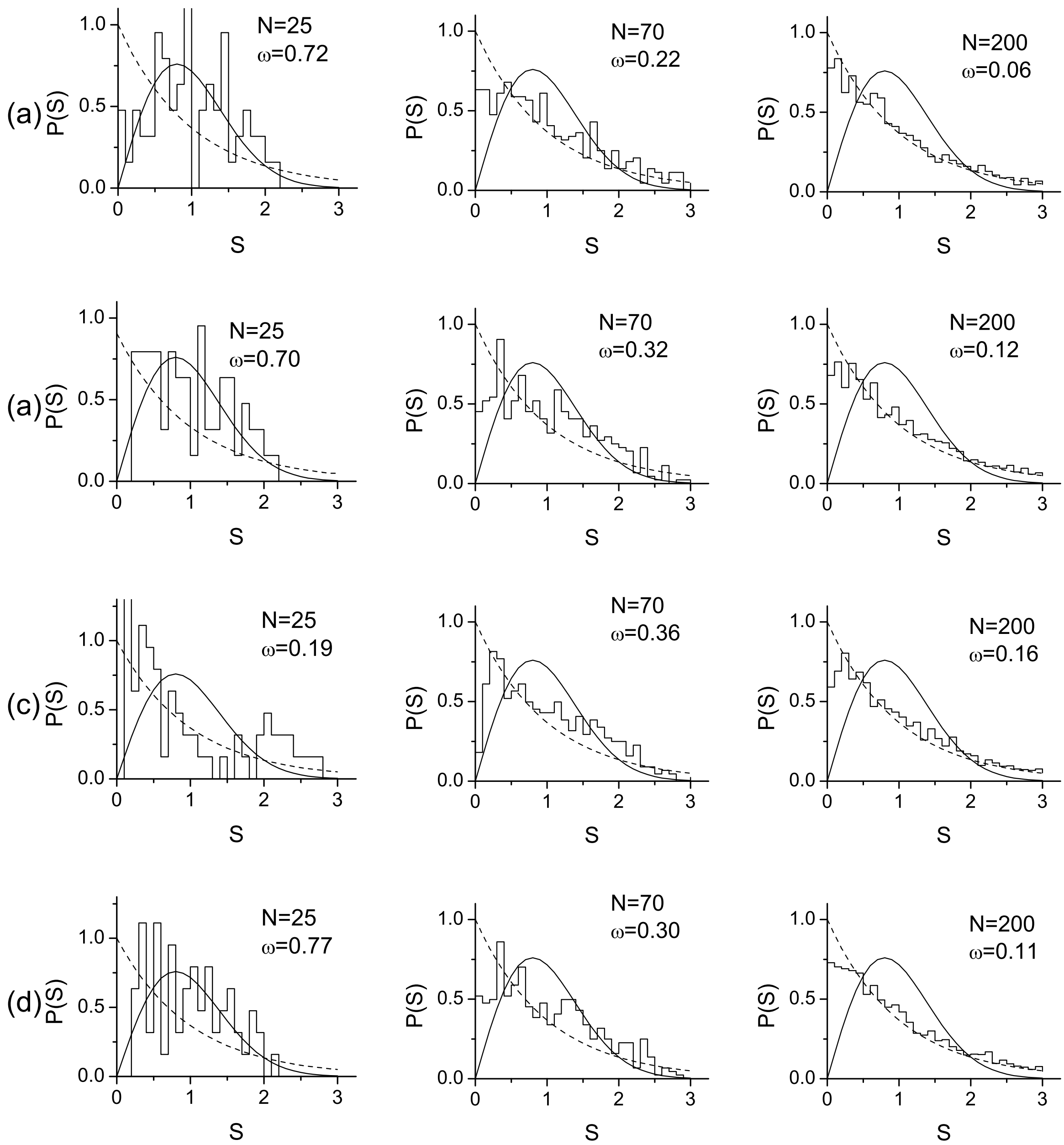


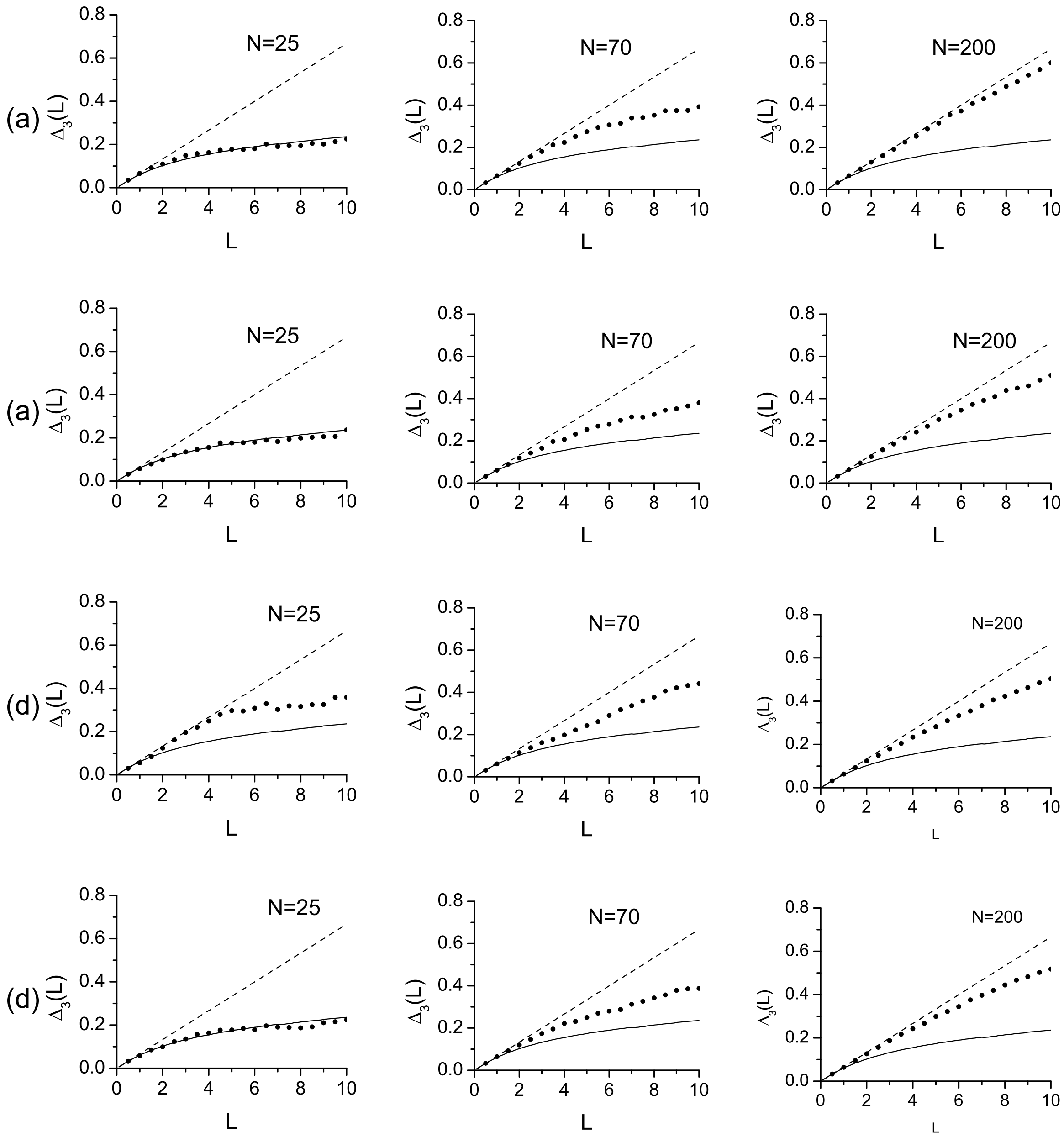
(a)

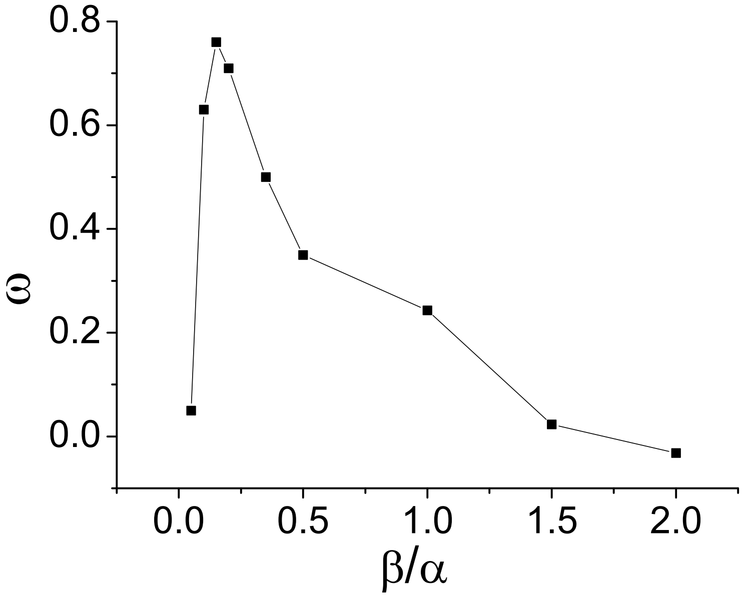


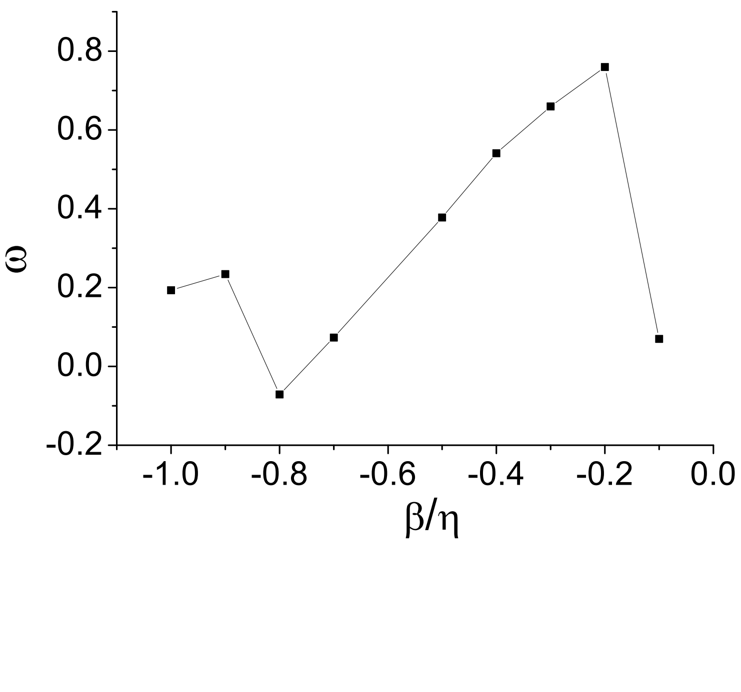
(b)

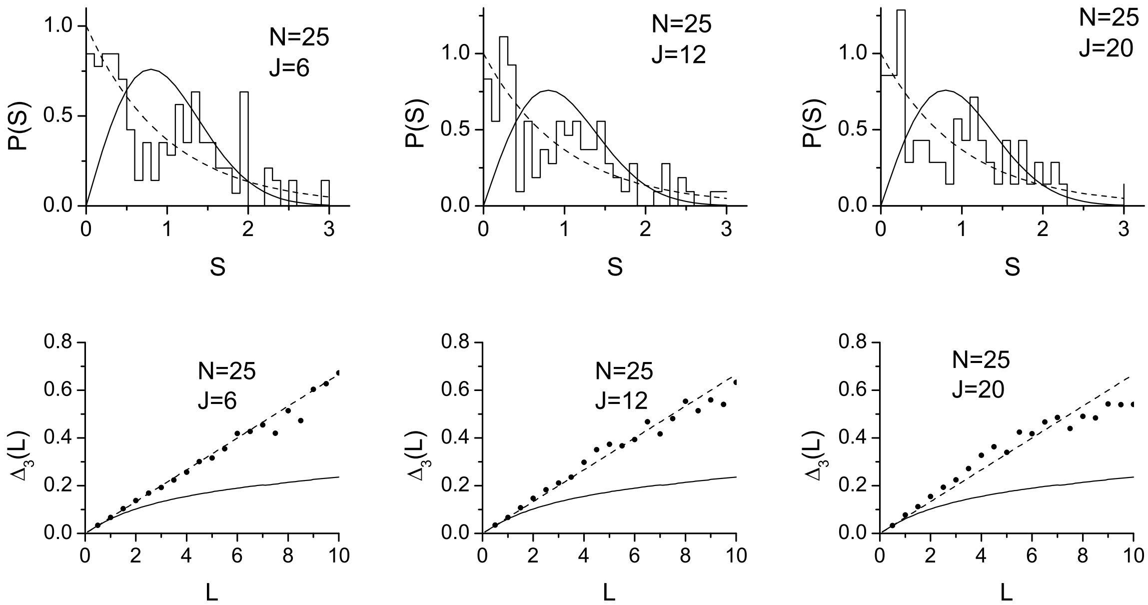

N=25
ω=0.64
N=70
ω=0.33
N=200
ω=0.13
P(S)
S
N=25
N=70
N=200
Δ3(L)
L

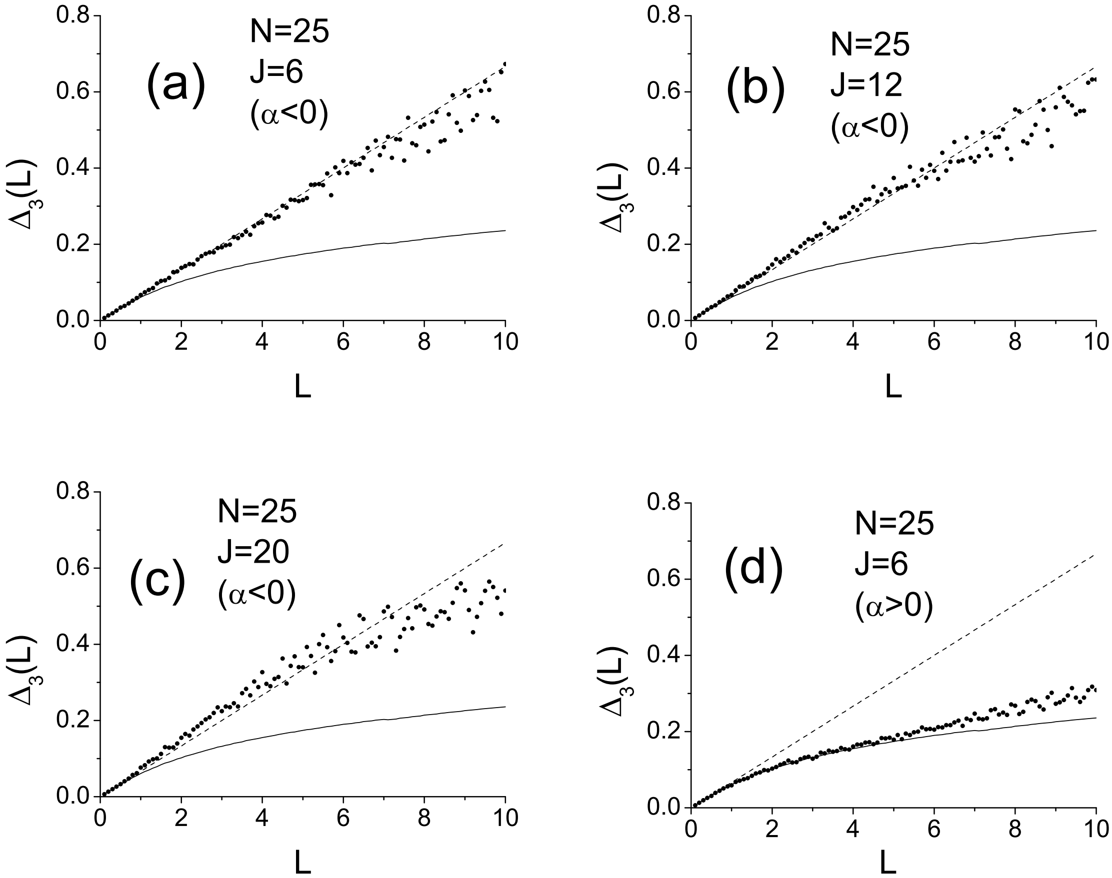

(a)
N=25
ω=0.72
N=70
ω=0.22
N=200
ω=0.06
(a)
N=25
ω=0.70
N=70
ω=0.32
N=200
ω=0.12
(c)
N=25
ω=0.19
N=70
ω=0.36
N=200
ω=0.16
(d)
N=25
ω=0.77
N=70
ω=0.30
N=200
ω=0.11
P(S)
S


(a)
N=25
N=70
N=200
(a)
N=25
N=70
N=200
(d)
N=25
N=70
N=200
(d)
N=25
N=70
N=200
$\Delta_3(L)$
L

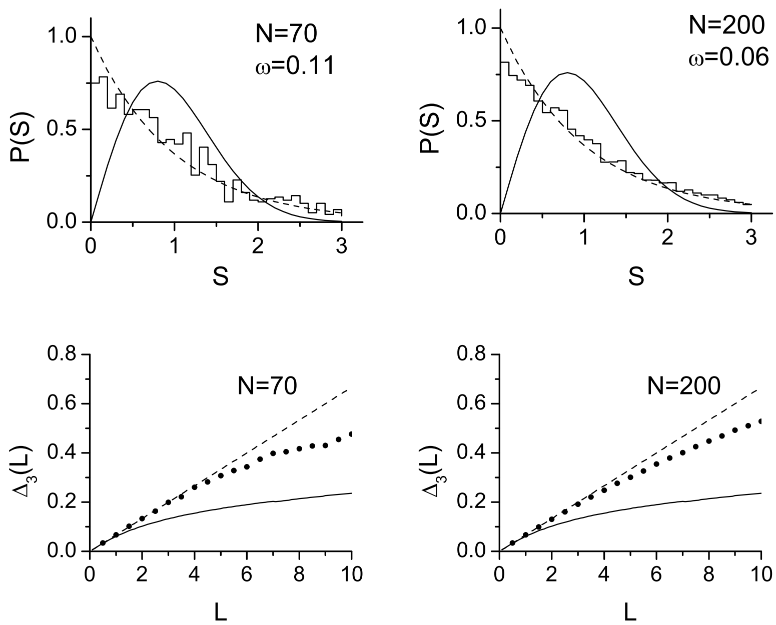

ω
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
β/α


ω
0.8
0.6
0.4
0.2
0.0
-0.2
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
β/η


N=25
J=6
N=25
J=12
N=25
J=20
P(S)
S
$\Delta_3(L)$
L



(a)
N=25
J=6
(α<0)
(b)
N=25
J=12
(α<0)
(c)
N=25
J=20
(α<0)
(d)
N=25
J=6
(α>0)
Δ3(L)
L
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
10


N=70
ω=0.11
P(S)
S
N=200
ω=0.06
N=70
Δ3(L)
L
N=200